

SEISMIC RESPONSES OF A 4-SPAN BRIDGE SYSTEM SUBJECTED  
TO MULTIPLE-SUPPORT GROUND EXCITATION

by

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ABSTRACT

Two different response components, i.e., the quasi-static response and the dynamic response, are used to model the dynamic behavior of a four-span continuous box girder bridge structure under multiple-support ground excitation. The input ground excitations considered include a periodic sine curve ground acceleration forcing function as well as an artificial strong ground motion record. Depending on the ground motion delay time, the response of multiple-support excitation is found to be sometimes larger than those of the traditional fixed-support analysis. This tends to suggest that the assumption of fixed-support ground excitation does not necessarily always lead to a conservative design of bridge structure.

INTRODUCTION

The seismic structural analysis of a bridge system is usually based on the assumption that all the supporting bridge piers and abutments are simultaneously excited by the earthquake ground motion. This assumption is valid only when the bridge pier spans are relatively small comparing to the horizontal propagating velocity of the seismic wave. Under such condition, there exists small and negligible relative ground displacements between piers, and only the bridge super-structure will be excited to have significant dynamic responses.

However in recent years, bridge designers worldwide tend to design longer span bridges due to aesthetic and construction considerations. In addition, the type of foundation and the surrounding soil condition of each pier maybe very different for an actual bridge system. Therefore, it is believed that the supports of a bridge structure might be subjected to different earthquake ground motions, whereas the usual assumption of simultaneous fixed-support excitation might not be valid.

Several studies have been conducted to investigate the effect of

multiple-support ground excitation on different structures. For example, Bofdanoff et al. (2) studied the influence of the transmission time of a seismic-like ground disturbance upon the response of a simple bridge model. The responses were evaluated in terms of extreme value statistics. Using the SRSS modal superposition technique, Show (10) attempted to analyze the multiple-support structural response by a series of single-support excitation solutions. By a similar approach, Wu et al. (12) proposed a multiple-support response spectrum method to compute the dynamic responses of typical piping structures in a nuclear power plant. Werner et al. (11) analyzed the three-dimensional response of a single-span bridge supported on an elastic half-space and subjected to incident shear waves. Based on random vibration methodology, Lee and Penzien (7) studied the cross-correlation between piping supports of a nuclear power facility. Recently, Dumanoglu and Severn (4) also examined the dynamic behavior of a doubly-encastered beam and a portal frame subjected to pulse accelerations.

Although the analytical approaches might be different, but most of the above-mentioned studies tend to suggest that the phase differences in the input ground motions applied to the structural foundations could have significant effects on the overall dynamic response. The objectives of this paper are: i) to investigate the dynamic behavior of a typical four-span bridge structure under multiple-support earthquake ground excitation; and ii) to assess the validity of the traditional assumption that all the bridge piers and abutments are excited simultaneously.

#### GOVERNING EQUATION OF MULTIPLE-SUPPORT EXCITED STRUCTURES

Based on the methodology originally proposed by Clough and Penzien (3), the governing motion equation can be derived for a structure under multiple-support ground excitation. Specifically, the total number of motion degree-of-freedom,  $n$ , can be divided into two parts: i.e., i)  $n_s$  degree-of-freedom corresponding to the supports, and ii) the remaining non-support  $n_b$  degree-of-freedom. Therefore, the general motion equation for a typical structural system under earthquake loads is expressed as follows:

$$\begin{pmatrix} M_s & M_{sb} \\ M_{sb}^T & M_b \end{pmatrix} \begin{pmatrix} \ddot{Y}_s \\ \ddot{Y}_b \end{pmatrix} + \begin{pmatrix} C_s & C_{sb} \\ C_{sb}^T & C_b \end{pmatrix} \begin{pmatrix} \dot{Y}_s \\ \dot{Y}_b \end{pmatrix} + \begin{pmatrix} K_s & K_{sb} \\ K_{sb}^T & K_b \end{pmatrix} \begin{pmatrix} Y_s \\ Y_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1)$$

whereas  $[\ddot{Y}_s]$ ,  $[\dot{Y}_s]$ , and  $[Y_s]$  are the absolute acceleration, velocity, and displacement of the non-support  $n_s$  degree-of-freedom.  $[M_s]$ ,  $[C_s]$ ,  $[K_s]$  are the associated mass, damping and stiffness matrices.  $[\ddot{Y}_b]$ ,  $[\dot{Y}_b]$ , and  $[Y_b]$  are the absolute acceleration, velocity, and displacement of the  $n_b$  support degree-of-freedom. Similarly,  $[M_b]$ ,  $[C_b]$ , and  $[K_b]$  are the corresponding mass, damping and stiffness matrices respectively.

Physically, the structural response due to multiple-support ground excitation can be interpreted as the combination of the following two response components: i) the "dynamic response component," i.e., the structural dynamic response relative to the ground due to fixed-support ground excitation; and ii) the "quasi-static response component," i.e.,

the absolute structural response induced by different support movements. Hence, the total structural response due to multiple-support ground excitation can be expressed as follows:

$$[Y] = \begin{pmatrix} Y_s \\ Y_b \end{pmatrix} = \begin{pmatrix} U_s^d \\ 0 \end{pmatrix} + \begin{pmatrix} Y_s^s \\ Y_b \end{pmatrix} \quad (2)$$

whereas  $[U_s^d]$  is the relative dynamic response of the non-support degree-of-freedom,  $[Y_b]$  is the absolute earthquake ground displacement at the supports, and  $[Y_s^s]$  is the quasi-static structural response.

Consider first the quasi-static response component of a multiple-support structure. Since there exists no external loading other than the relative ground displacement, the structure behaves statically. Therefore, the inertia force and the damping force as expressed in Eq. 1 can be neglected. This leads to a simple static equilibrium equation:

$$\begin{pmatrix} K_s & K_{sb} \\ K_{sb}^T & K_b \end{pmatrix} \begin{pmatrix} Y_s^s \\ Y_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3)$$

For given relative ground displacement  $[Y_b]$ , the only unknown in the above equation is  $[Y_s^s]$ , which can easily be expressed as follows:

$$[Y_s^s] = -[K_s]^{-1}[K_{sb}][Y_b] = [R][Y_b] \quad (4)$$

The matrix  $[R]$ , defining  $[R] = -[K_s]^{-1}[K_{sb}]$ , can be interpreted as the "influence matrix" of non-support degree-of-freedom due to support ground displacement.

By replacing Eq. 2 into Eq. 1, the following governing motion equation for the dynamic response component of a multiple-support excited structure is derived:

$$\begin{aligned} [M_s][\ddot{U}_s^d] + [C_s][\dot{U}_s^d] + [K_s][U_s^d] = & -([M_s][\ddot{Y}_s^s] + [M_{sb}][\ddot{Y}_b]) \\ & + [C_s][\dot{Y}_s^s] + [C_{sb}][\dot{Y}_b] + [K_s][Y_s^s] + [K_{sb}][Y_b] \end{aligned} \quad (5)$$

From Eq. 4,  $[\dot{Y}_s^s] = [R][\dot{Y}_b]$ , and  $[\ddot{Y}_s^s] = [R][\ddot{Y}_b]$ . Substituting these equations and Eq. 3 into Eq. 5, it becomes:

$$\begin{aligned} [M_s][\ddot{U}_s^d] + [C_s][\dot{U}_s^d] + [K_s][U_s^d] = & -([M_s][R] + [M_{sb}])[R][\ddot{Y}_b] \\ & -([C_s][R] + [C_{sb}]][R][\dot{Y}_b] \end{aligned} \quad (6)$$

In the above equation, the first term on the right hand side corresponds to the inertia force due to support acceleration  $[R][\ddot{Y}_b]$ , and the second term is the damping force due to support ground velocity  $[R][\dot{Y}_b]$ . Since this damping force equals zero for stiffness proportional damping and is relatively small for other forms of damping, therefore, it can be neg-

lected (8). In addition, assuming the structural system is modeled as a lumped-mass model, the system mass matrix  $[M]$  reduces to a diagonal matrix, i.e.,  $[M_{sb}]$  is a null matrix. Hence, Eq. 6 becomes:

$$[M_s][\ddot{U}_s^d] + [C_s][\dot{U}_s^d] + [K_s][U_s^d] = - [M_s][R][\ddot{Y}_b] \quad (7)$$

Eq. 7 is very similar to the traditional motion equation of a structural system subjected to fixed-support ground excitation. Except that in this multiple-support excitation condition, the inertia force due to ground acceleration must be modified by the influence matrix  $[R]$ . Notice that  $[R]$  is a  $n_s \times n_b$  matrix, whereas  $[\ddot{Y}_b]$  is a  $n_b \times 1$  matrix.

For given earthquake ground accelerations of the supports  $[\ddot{Y}_b]$ , the dynamic response component of the multiple-support system can be computed from Eq. 7. By combining the dynamic response component and the quasi-static response component (Eq. 3), the total structural response of the system under multiple-support ground excitation can be obtained. Based on the above-mentioned methodology, the computer program NEABS (Nonlinear Earthquake Analysis of Bridge System) as developed by Penzien et al. (9) has been modified. Using the program, the dynamic behavior of a typical four-span bridge structure under multiple-support earthquake ground excitation has been evaluated. The results are presented next.

#### EXAMPLE BRIDGE UNDER SINUSOIDAL GROUND EXCITATION

A typical four-span continuous reinforced concrete box-girder bridge structure has been incorporated in this study. As shown in Fig. 1, the bridge has a total span of 136 meters with two 28 meters end spans and two 40 meters center spans. The height of the three center bridge piers is 8 meters with 1.6 meters diameter. Details of the box-girder bridge deck are also shown in Fig. 1. In accordance with the design practice in the Republic of China, the connections between the deck and the three supporting piers are assumed to be hinged in all directions. The abutments at the ends are assumed to be roller-connected with the bridge deck in the longitudinal direction. They are partially restrained against translation and rotation in the transverse direction.

To better understand the dynamic characteristics of the example bridge, the periods of the modes of vibration and the associated modal shapes have first been computed and are plotted in Fig. 2. As shown in the figure, the fundamental mode of vibration is in the x-direction (longitudinal) with period equals 0.782 sec. The second mode is in the z-direction (transverse) with period equals 0.699 sec, whereas the third mode is in the y-direction and the natural period is equal to 0.506 sec. The characteristics of the other three higher modes are also plotted in the figure. For details, the reader is referred to Lai et al. (6).

In this investigation of the structural response of the example bridge subjected to earthquake loads, the sinusoidal ground acceleration forcing function has first been used for simplicity purposes. Fig. 3-a depicts that the three center piers are being excited with the traveling sinusoidal seismic wave in the longitudinal direction. In a similar way, Fig. 3-b shows that the two end abutments and the three center piers are

being excited with the traveling wave in the transverse direction. Denoting the predominant period of the sinusoidal acceleration forcing function as  $T_g$ , it is assumed to be equal to 0.48 sec. The maximum acceleration is equal to 0.3g, with maximum ground displacement equals 1.72 centimeters. Assuming the arrival time of the incident sinusoidal wave of each pier is a function of  $T_g$ , then depending on the distances between bridge piers, the delay time lags can be expressed as fractions of  $T_g$ . This is illustrated in Fig. 3. Therefore, by varying the delay time lag between piers, the sensitivity of the structural response of the example bridge under multiple-support sinusoidal acceleration can be investigated.

Using the modified computer program NEABS, the dynamic response has been computed for the example bridge under both longitudinal and transverse direction sinusoidal ground excitation. Consider first the longitudinal direction, the resulting maximum bending moments (in absolute values) at the bottoms of the three center bridge piers, i.e.,  $P_1$ ,  $P_2$ , and  $P_3$ , are plotted in Figs. 5-a, -b, and -c respectively. Notice that the dynamic response components of the three piers (as shown in dashed lines) are almost the same. This can be explained by the fact that the bridge deck is relatively stiff in the longitudinal direction, thus, the bridge behaves as a rigid body under ground excitation in that direction. When the quasi-static response components are included, the total responses are found to vary with different delay time lag. Nevertheless, the largest response occurs when the delay time lag is equal to zero, i.e., the bridge is under simultaneous fixed-support ground excitation.

Figs. 5-d, -e, and -f present the results of transverse directional multiple-support sinusoidal excitation. As shown in the figures, when the delay time lag is equal to  $1/2T_g$ , the maximum bending moments at the bottoms of the two outer piers  $P_1$  and  $P_3$  are approximately 36% to 100% greater than those of the fixed-support excitation. In addition, the total bridge response varies with different earthquake delay time lag, and no specific trend can be detected.

#### EXAMPLE BRIDGE UNDER ARTIFICIAL STRONG GROUND MOTION

To further investigate the seismic behavior of the example multiple-support bridge structure, an artificial strong ground motion with 10 secs duration has been incorporated in this study. As plotted in Fig. 4, the acceleration time history was obtained by matching the bridge design response spectrum as proposed by the Applied Technology Council (ATC-6) (1). The peak ground acceleration is equal to 0.3g, whereas the peak ground displacement equals 7.26 centimeters. For details, the reader is referred to Lai et al. (5).

For different ground motion delay time lag, the dynamic response of the example bridge under multiple-support artificial ground excitation has been computed for both longitudinal and transverse directions. The delay time lag unit,  $T_g$ , has been changed to 0.782 sec, which is the same as the fundamental natural period of the example four-span bridge system. Under longitudinal excited ground motion, the resulting maximum bending moments at the bottoms of the three supporting piers are presented in Figs. 6-a, -b, and -c respectively. Similar with the results discussed

in the preceding section, the dynamic response components of the three center piers are almost the same. By combining the quasi-static response components, the total responses of the three piers are significantly different. Depending on the delay time lag, the total response is greater than those obtained by the fixed-support assumption by approximately 10% to 75%. In particular, the maximum bending moment of the center pier  $P_2$  behaves very irregular with varying delay time lag.

Under transverse excited artificial strong ground motion, the resulting maximum bottom bending moments of the three center piers are shown in Figs. 6-d, -e, and -f respectively. The results indicate that the contribution of quasi-static response component increases with increasing delay time lag beyond  $1T$ . Therefore, the total response tends to increase with increasing delay time lag. As an example, the maximum bottom bending moment of  $P_3$  is a relative minimum when the delay time lag equals  $1T$ , and it increases with the delay time lag. When the delay time lag is equal to  $2(3/4)T$ , the total response of bottom moment of  $P_3$  is greater than that of the fixed-support excitation by approximately 70%.

#### CONCLUSIONS

In this paper, two different response components, i.e., the quasi-static response and the dynamic response, have been devised to model the dynamic behavior of bridge systems under multiple-support ground excitations. Specifically, the seismic response of a four-span continuous reinforced concrete box girder bridge structure has been analyzed. The input ground excitations considered include a periodic sine curve ground acceleration forcing function and an artificial strong ground motion record. Depending on the ground motion delay time lag, the response of multiple-support excitation is found to be sometimes much larger than those obtained by the traditional fixed-support analysis. This tends to suggest that the usual assumption of simultaneous fixed-support ground excitation does not necessarily lead to a conservative design of bridge systems.

#### ACKNOWLEDGEMENTS

This research was supported by the Chinese National Science Council under Grant NSC72-414-P011-02. The author acknowledges Mr. Y.J. Lee for his aid in the computer programming and calculations. The assistance of the author's colleagues at the National Taiwan Institute of Technology is also greatly appreciated.

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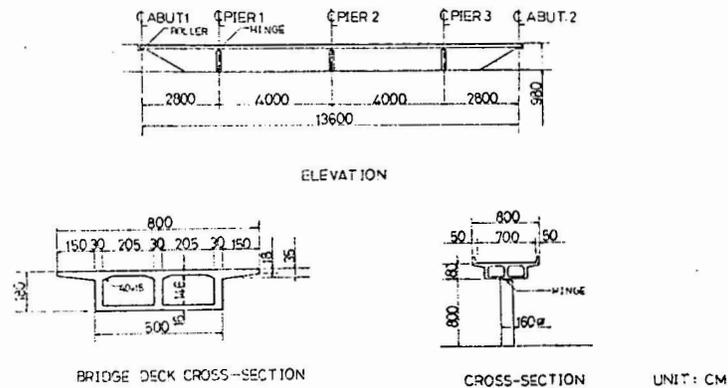


Fig. 1 Elevation & Cross-Sections of Example 4-Span Bridge

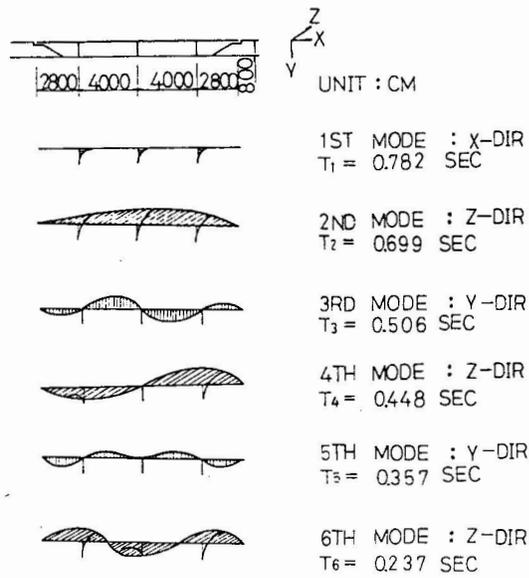


Fig. 2 Dynamic Characteristics of Example Bridge

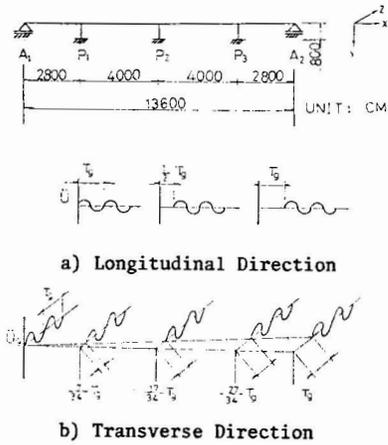


Fig. 3 Multiple-Support Sinusoidal Ground Excitations

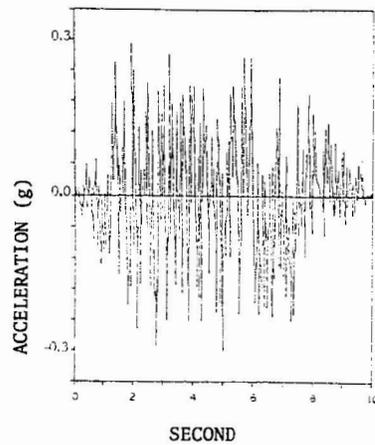
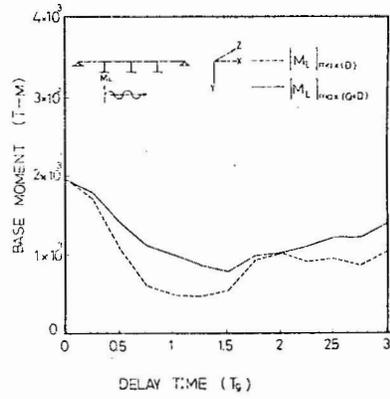
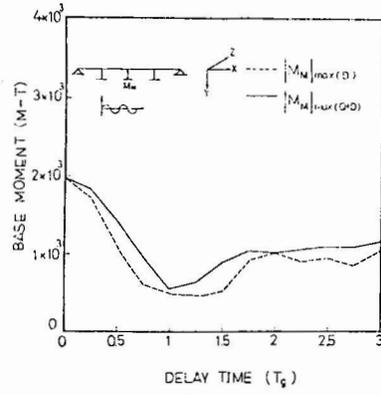


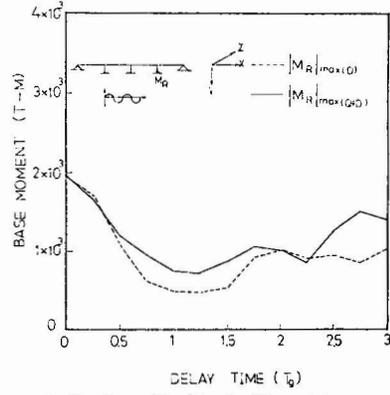
Fig. 4 Artificial Strong Ground Motion Record



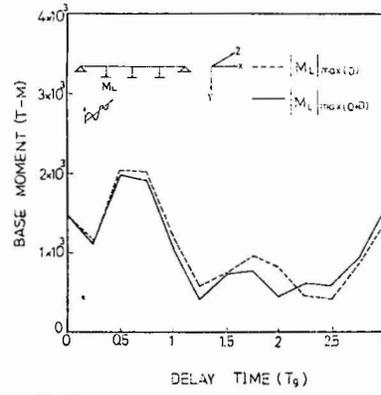
a) P<sub>1</sub> Longitudinal Direction



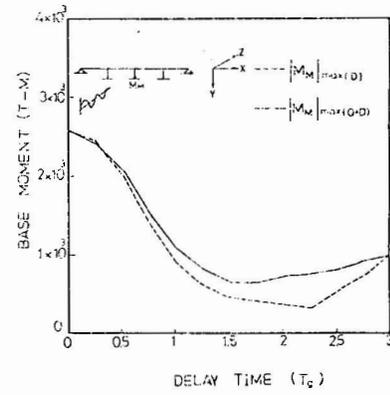
b) P<sub>2</sub> Longitudinal Direction



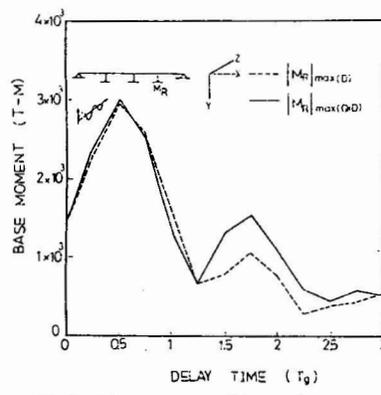
c) P<sub>3</sub> Longitudinal Direction



d) P<sub>1</sub> Transverse Direction

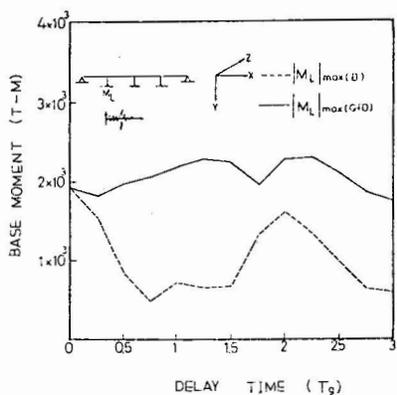


e) P<sub>2</sub> Transverse Direction

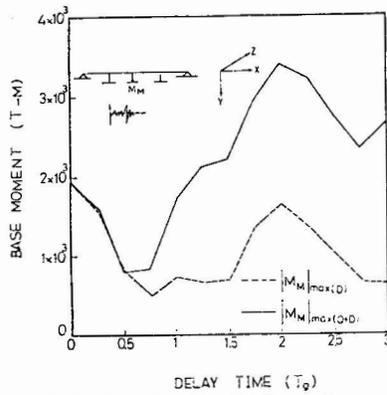


f) P<sub>3</sub> Transverse Direction

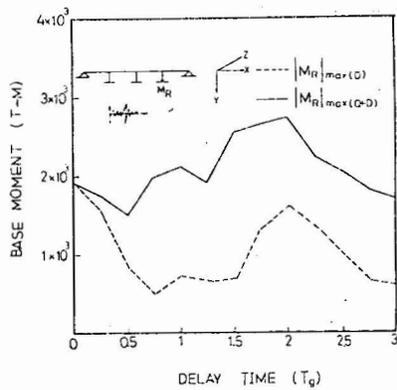
Fig. 5 Seismic Responses of Example Bridge Under Multiple-Support Sinusoidal Ground Excitations



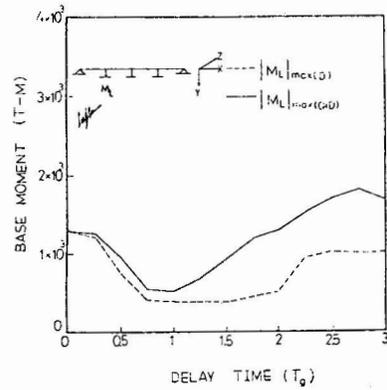
a) P<sub>1</sub> Longitudinal Direction



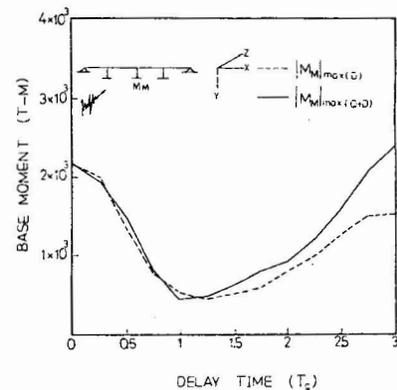
b) P<sub>2</sub> Longitudinal Direction



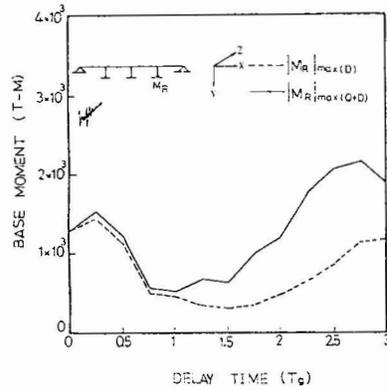
c) P<sub>3</sub> Longitudinal Direction



d) P<sub>1</sub> Transverse Direction



e) P<sub>2</sub> Transverse Direction



f) P<sub>3</sub> Transverse Direction

Fig. 6 Seismic Responses of Example Bridge Under Multiple-Support Artificial Strong Ground Motion